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THE SIMULATION OF THREE-DIMENSIONAL WAVE PROPAGATION BY A SCALAR TLM MODEL

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ABSTRACT

This paper presents a novel scalar transmission line matrix simulating the propagation of a Hertzian potential in three-dimensional space. The new method is numerically more efficient than the traditional TLM method.

INTRODUCTION

In the traditional Transmission Line Matrix (TLM) Method of numerical analysis [1], the propagation of all six components of the electromagnetic field is simulated by the propagation of corresponding voltage and current impulses in a complicated hybrid transmission line mesh. This leads to considerable memory and cpu time requirements. In order to reduce these requirements we propose the TLM simulation of Hertzian potentials from which the field components as well as the eigenvalues of resonant systems can be obtained. Since Hertzian potentials satisfy the scalar wave equation, only a scalar TLM network is required for three-dimensional simulations.

ANALYSIS OF THE SCALAR TLM NETWORK

Consider a unit cell of the new three-dimensional network which consists of a two-dimensional TLM network to which additional transmission lines are connected orthogonally at each junction as shown in Fig. 1. Since the dimensions of a unit cell must be

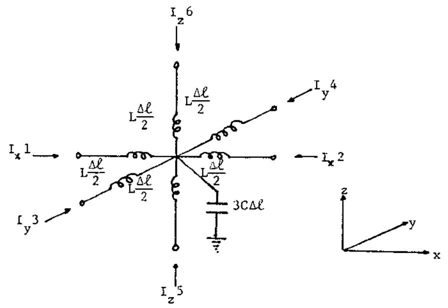

Fig. 1 - A unit cell of the three-dimensional scalar TLM network.

small compared with the wavelength, the voltages at the six ports are effectively equal to the voltage at the node. Furthermore, let us assume that there is no coupling between the lines. Under these assumptions, the following approximate first-order equations are obtained:

$$\nabla \vec{I} = -3\,C\,\frac{\partial V}{\partial t} \qquad (1a)$$

$$\nabla V = -L\,\frac{\partial \vec{I}}{\partial t} \qquad (1b)$$

where $\vec{I} = I_x\hat{x} + I_y\hat{y} + I_z\hat{z}$

and $I_x = I_x^1 - I_x^2$

$I_y = I_y^3 - I_y^4$

$I_z = I_z^5 - I_z^6$

These coupled equations may be uncoupled, yielding the following second-order differential equations in V and $\vec{I}$:

$$\{\nabla^2 - 3\,LC\,\frac{\partial^2}{\partial t^2}\}\;V = 0 \qquad (2a)$$

$$\{\nabla^2 - (\nabla x\nabla x) - 3\,LC\,\frac{\partial^2}{\partial t^2}\}\;\vec{I} = 0 \qquad (2b)$$

Note that although equation (2b) may not look like a wave equation, the second term involving the double curl of $\vec{I}$ is zero as a consequence of (1b). Finally, V as well as each component of $\vec{I}$ satisfy the following scalar wave equation:

$$(\nabla^2 - 3\,LC\,\frac{\partial^2}{\partial t^2})\;\psi\,(\vec{x},t) = 0 \qquad (3)$$

The propagation of this scalar wave can be easily simulated on the scalar three-dimensional TLM network using the same algorithms as the conventional TLM method.

NUMERICAL SIMULATION OF A SCALAR WAVE

The scalar wave function ψ can represent either a field component or a Hertzian potential. The physical nature of ψ determines its behaviour at boundaries. For instance, ψ will be subject to a reflection coefficient of -1 at a lossless electric wall if it represents a tangential electric or a normal magnetic field. A normal electric or a tangential magnetic field will be reflected with a coefficient of +1 in the same circumstances.

Once the boundary conditions are properly determined, the impulse response of the scalar network is found by iteration exactly as in the case of the classical two-dimensional TLM procedure.

The scattering matrix of a three-dimensional node in the scalar network is:

$$[S] = \frac{1}{3}\begin{bmatrix} -2 & 1 & 1 & 1 & 1 & 1 \\ 1 & -2 & 1 & 1 & 1 & 1 \\ 1 & 1 & -2 & 1 & 1 & 1 \\ 1 & 1 & 1 & -2 & 1 & 1 \\ 1 & 1 & 1 & 1 & -2 & 1 \\ 1 & 1 & 1 & 1 & 1 & -2 \end{bmatrix} \quad (4)$$

INHOMONGENEOUS STRUCTURES

Media with a dielectric constant ε_r different from unity can be simulated by loading all nodes inside these media with a capacitive stub. If the normalized characteristic admittance of the stub is y_o, the phase velocity and the scattering matrix of a stub-loaded node inside the medium will become

$$v_{ph} = c/\sqrt{3(1 + y_o/6)} \quad (5)$$

and

$$[S] = \frac{1}{6+y_o}\begin{bmatrix} -(y_o+4) & 2 & 2 & 2 & 2 & 2 & 2y_o \\ 2 & -(y_o+4) & 2 & 2 & 2 & 2 & 2y_o \\ 2 & 2 & -(y_o+4) & 2 & 2 & 2 & 2y_o \\ 2 & 2 & 2 & -(y_o+4) & 2 & 2 & 2y_o \\ 2 & 2 & 2 & 2 & -(y_o+4) & 2 & 2y_o \\ 2 & 2 & 2 & 2 & 2 & -(y_o+4) & 2y_o \\ 2 & 2 & 2 & 2 & 2 & 2 & y_o-6 \end{bmatrix} \quad (6)$$

The additional element of [S] represents the pulse on the stub line. As in the conventional TLM simulation, the capacitive loading of the nodes reduces the phase velocity and the wave impedance in the medium, and it automatically satisfies the interface condition when simulating, for example, an E-field component which is tangential, or a magnetic-type Hertzian potential which is normal to the dielectric interface. Further studies are required for the simulation of E-field components which are normal to such a dielectric interface (or an electric-type Hertzian potential normal to that interface). Such a simulation requires the introduction of a correction factor which is different from that used in the conventional TLM procedure.

APPLICATION TO RECTANGULAR RESONATORS

In order to demonstrate the scalar TLM performance, the first few eigenvalues of two rectangular cavities (one empty and the other dielectric slab loaded) were computed and then compared with exact analytical results. The empty resonator (Fig. 2) was analyzed with both the conventional and the scalar TLM method using the same mesh size and the same number of iterations. Table I compares the results and shows that the scalar method is more accurate (and seven times faster than the conventional one). Furthermore, memory size required is four times smaller for the scalar method.

The slab-loaded resonator (Fig. 3) was analyzed by simulating the propagation of a magnetic-type Hertzian potential normal to the slab-air interfaces. Table II compares the results with analytical results obtained by solving the transverse resonance condition for the first two LSE-modes.

Ref. [1] S. Akhtarzad, P.B. Johns, Proc. IEEE, vol. 122, no. 12, pp. 1344-48, Dec. 75.

DISCUSSION AND CONCLUSION

The classical three-dimensional TLM method of analysis can be simplified by simulating only one field component or a Hertzian potential in a scalar network. Such a network could be realized by a cubic array of coaxial lines shunt-connected at the nodes. The voltage propagating in such a network satisfies the scalar homogeneous wave equation. Boundary conditions at walls or interfaces depend on the physical nature of the simulated wave quantity.

Two examples demonstrate the performance of the method and compare it with the conventional TLM-method. Furthermore, all methods proposed to reduce time and memory requirements such as variable mesh size and window optimization can be applied to the scalar method.

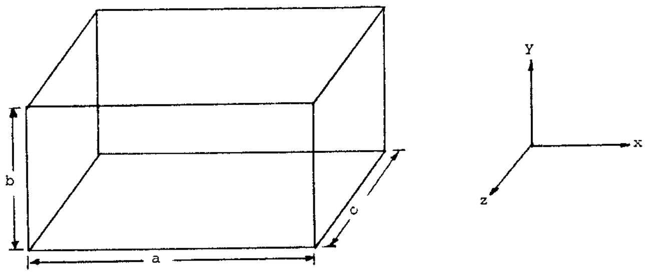

Fig. 2 - Empty Cavity evaluated with the scalar T.L.M.-method ($a = 12\ \Delta\ell$, $b = 6\ \Delta\ell$, $c = 8\ \Delta\ell$)

	ANALYTIC VALUE	SCALAR T.L.M.	Δ %	CONVENTIONAL T.L.M.	Δ %
TE_{101}	0.07511	0.07480	0.4	0.07480	0.4
TM_{110}	0.09317	0.09299	0.18	0.09249	0.72
TM_{201}	0.10417	0.10379	0.36	0.10337	0.76
TE_{111}	0.11219	0.11199	0.18		
TM_{210}	0.11785	0.11759	0.22		

Table I - Comparison of the first six normalized resonant frequencies of a homogeneously filled rectangular cavity with values obtained by the scalar and the conventional TLM methods.

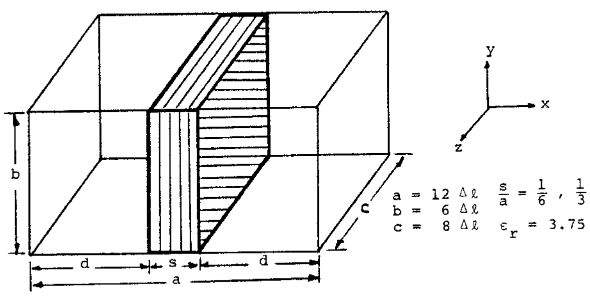

Fig. 3 - Dielectric loaded cavity evaluated with scalar T.L.M.

MODE	S/A	ANALYTIC VALUE	SCALAR T.L.M.	ERROR(%)
LSE_{101}	1/6	0.0522	0.0516	1.15
	1/3	0.0445	0.0440	1.12
LSE_{201}	1/6	0.0988	0.0935	0.3
	1/3	0.0776	0.0759	2.19

Table II - First two LSE modes evaluated with the scalar T.L.M. (Normalized Resonant Frequencies)